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NASA TM-X-69353

X-300-71-1
NTIS HC 83,75

**AN ERROR ANALYSIS
OF THE
RECOVERY CAPABILITY
OF THE
RELATIVE SEA-SURFACE PROFILE
OVER THE
PUERTO RICAN TRENCH
FROM
MULTI-STATION
AND
SHIP TRACKING
OF
GEOS-II**



WALLOPS STATION

WALLOPS ISLAND, VIRGINIA

(NASA-TM-X-69353) AN ERROR ANALYSIS OF
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TRACKING (NASA) 33 p HC \$3.75 CSCL 08C

N73-16347

Unclas
52733

G3/13

MAY 1971

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by

H.R. Stanley

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Wallops Island, Virginia

C.F. Martin
N.A. Roy
J.R. Vetter

WOLF RESEARCH AND DEVELOPMENT CORPORATION
6801 Kenilworth Avenue
Riverdale, Maryland 20840

Contract NAS6-1942

May 1971

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ABSTRACT

An error analysis has been performed to examine the height error which might be expected in a relative sea-surface profile over the Puerto Rican Trench as determined by a combination of land-based multi-station C-Band radars and optical lasers and one ship-based radar tracking the GEOS-II Satellite. It has been shown that two relative profiles can be obtained: one profile using available South-to-North passes of the satellite and one profile using available North-to-South type passes. An analysis of multi-station tracking capability has determined that only Antigua and Grand Turk radars are required to provide satisfactory orbits for South-to-North type satellite passes, while a combination of Merritt Island, Bermuda and Wallops radars provide secondary (back-up) orbits for North-to-South passes.

In addition, analysis of ship tracking capabilities has shown that high elevation single pass range-only solutions are necessary to give only moderate sensitivity to systematic error effects.

However, range only solutions utilizing two satellite passes provide a much reduced sensitivity to exact pass geometries. A tracking schedule is presented for the months of June and July 1970 which offers 35 opportunities during the 60 day period for determination of profile points utilizing only South-to-North satellite passes and the Antigua, Grand Turk and ship tracking stations.

SECTION 1.0

INTRODUCTION

The C-Band radar network and optical lasers have provided precision tracking of the GEOS-II satellite since its launch in January 1968. Because of their precision orbit determination capability a variety of experiments have been performed using single station and multi-station tracking on both long and short arcs of GEOS-II. An experiment which is currently in the mission planning stage is to track GEOS-II to determine the relative sea-surface height profile over the Puerto Rican Trench by utilizing C-Band multi-station land-based C-Band radars and optical lasers if required.

The experiment rationale accepts the theory that the sea-surface everywhere approximates the geoid and that determination of the height of a ship, relative to the spheroid, at various points as the ship traverses a gravity anomaly such as the Puerto Rican Trench will recover the relative geoid profile of the anomaly.

This study will therefore concern itself with illustrating the effects of various systematic errors which may be encountered during such an experiment and determination of the optimum geometry and procedures to be utilized. Specifically the following factors will be investigated:

- 1) Assuming the availability of tracking support from the stations listed in Table 1, determine the optimum station configuration for best ship height determination.
- 2) Assuming the continuous availability of the GEOS-II satellite for the period of 1 June

TABLE 1
SAO C-5 DATUM STATION POSITIONS

<u>Station</u>	<u>North Latitude</u>	<u>East Longitude</u>	<u>Height Above Spheroid (meters)</u>
ANTIGUA Radar	17° 8' 37"235	298° 12' 25"603	13.086
GRAND TURK Radar	21° 27' 45"339	288° 52' 4"055	-16.908
BERMUDA Radar	32° 20' 52"456	295° 20' 46"054	-39.775
MERRITT Radar	28° 25' 28"894	279° 20' 7"380	-41.768
WALLOPS Radar	37° 51' 36"353	284° 29' 25"849	-42.262
CURACO Laser	12° 5' 25"684	291° 9' 44"086	-50.500
P. RICO Laser	18° 30' 0"0	292° 50' 0"0	10.000
SHIP Radar #1	20° 40' 0"0	293° 42' 0"0	15.000
SHIP Radar #2	20° 00' 0"0	293° 42' 0"0	15.000
SHIP Radar #3	19 40' 0"0	293° 42' 0"0	15.000
SHIP Radar #4	19 20' 0"0	293° 42' 0"0	15.000
SHIP Radar #5	19 00' 0"0	293° 42' 0"0	15.000
SHIP Radar #6	18 40' 0"0	293° 42' 0"0	15.000
SHIP Radar #7	18° 20' 0"0	293° 42' 0"0	15.000

through 30 July 1970 as shown in Tables 2 and 3, determine the optimum orbit geometry and length and the recommended tracking schedule.

- 3) Determine the type of ship track necessary.
- 4) Evaluate such critical factors as ship range bias and ground station position errors and range biases.

As an aid in understanding the problem and for purposes of illustration Figure 1 presents the the geometry of the test area and Figure 2 presents an approximate bottom profile of the Puerto Rican Trench. Also shown in Figure 2 (reproduced from reference [1]) is one determination of the geoidal separation between a reference spheroid and the geoid over the Trench.

To implement this study an orbital error analysis computer program (ORAN) was utilized. The following sections of this report first describe the measurement model used for this investigation (measurement standard deviations) and second, provides a discussion of the various other (unmodeled) errors which may be expected but are not normally considered in the measurement model error analysis. Next, an analysis is presented which defines the optimum tracking station configuration and is followed by discussions concerning the relative merits of ship height estimation using one satellite pass vs two satellite passes. Finally, the results and conclusions of this study are presented along with recommendations concerning the optimum tracking schedule and items which warrant further study.

Hour	Day		Day		Day		Day		Day	
0	S/N 50° W	1	S/N 54° W	7	S/N 57° W	13	S/N 63° W	19	S/N 67° W	25
4										
8										
12	N/S 67° E		N/S 65° E		N/S 53° E		N/S 51° E		N/S 45° E	
16										
20										
24	S/N 40° E		S/N 38° E		S/N 34° E		S/N 31° E		S/N 28° E	
4	S/N 38° W	2	S/N 41° W	8	S/N 44° W	14	S/N 48° W	20	S/N 50° W	26
8										
12	N/S 72° W		N/S 79° W		N/S 78° E		N/S 64° E		N/S 66° E	
16										
20										
24	S/N 53° E		S/N 49° E		S/N 45° E		S/N 42° E		S/N 37° E	
4	S/N 28° W	3	S/N 30° W	9	S/N 33° W	15	S/N 35° W	21	S/N 38° W	27
8										
12	N/S 59° W		N/S 58° W		N/S 71° W		N/S 74° W		N/S 73° W	
16										
20										
24	S/N 69° E		S/N 61° E		S/N 59° E		S/N 53° E		S/N 50° E	
4	S/N 20° W	4	S/N 22° W	10	S/N 24° W	16	S/N 26° W	22	S/N 28° W	28
8										
12	N/S 21° E									
16	N/S 41° W		N/S 46° W		N/S 47° W		N/S 56° W		N/S 59° W	
20										
24	S/N 76° W		S/N 82° E		S/N 75° E		S/N 69° E		S/N 66° E	
4		5		11		17		23	S/N 20° W	29
8										
12	N/S 30° E		N/S 28° E		N/S 24° E		N/S 22° E			
16	N/S 29° W		N/S 32° W		N/S 36° W		N/S 38° W		N/S 45° W	
20	S/N 21° E									
24	S/N 70° W									
4		6	S/N 76° W	12	S/N 74° W	18	S/N 88° W	24	S/N 74° E	30
8										
12	N/S 40° E		N/S 39° E		N/S 36° E		N/S 31° E		N/S 29° E	
16	N/S 21° W		N/S 23° W		N/S 25° W		N/S 28° W		N/S 31° W	
20	S/N 28° E									
24			S/N 26° E		S/N 23° E		S/N 21° E			

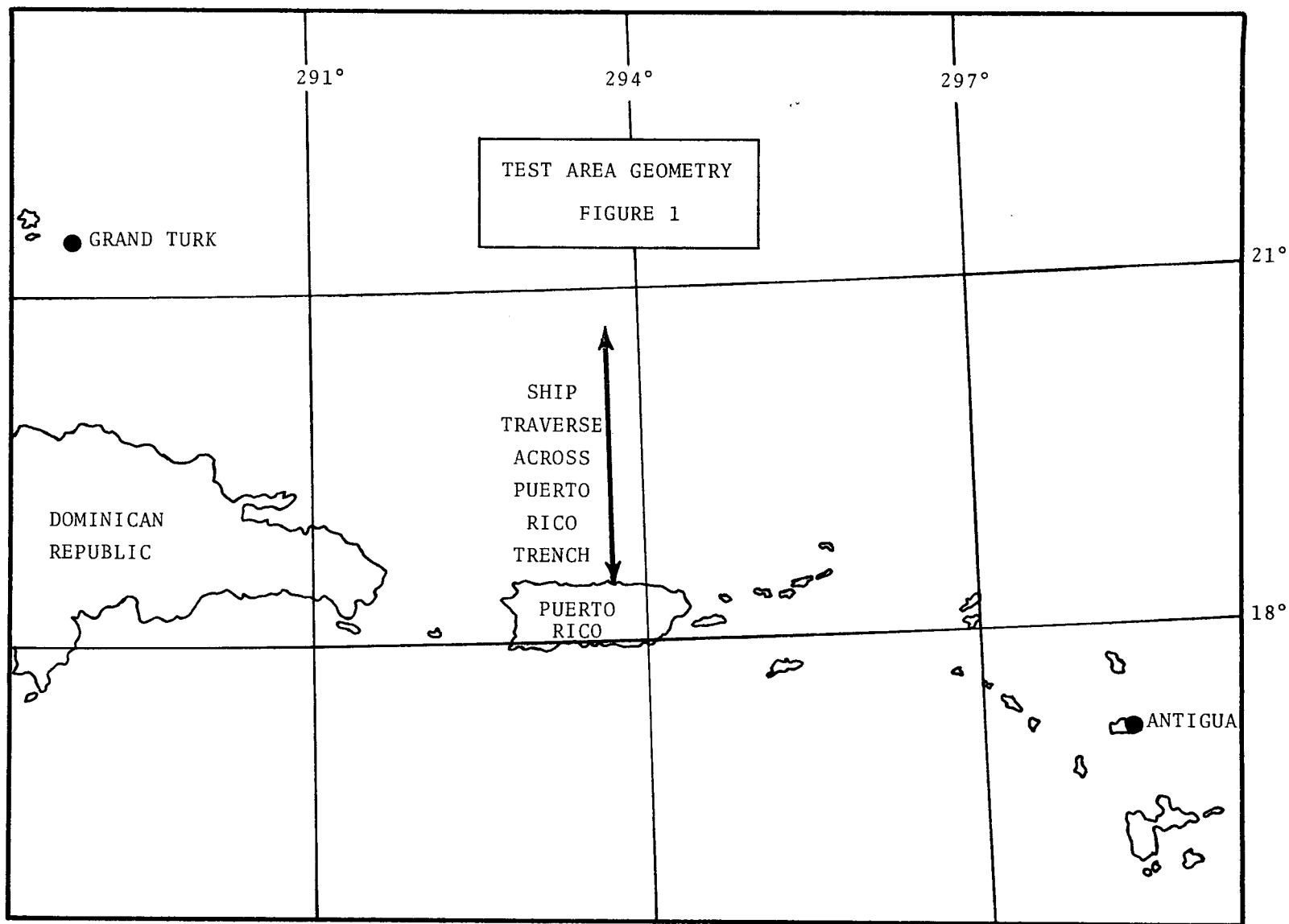
Table 2

Puerto Rico Trench Tracking Schedule For June 1970

Hour	Day		Day		Day		Day		Day	
0	1	S/N 72° W	7	S/N 76° W	13	S/N 80° W	19	S/N 81° W	25	S/N 78° E
4										
8										
12		N/S 40° E		N/S 37° E		N/S 38° E		N/S 31° E		N/S 29° E
16		N/S 22° W		N/S 25° W		N/S 28° W		N/S 30° W		N/S 35° W
20	2		8		14		20		26	
24		S/N 25° E		S/N 23° E						
4		S/N 55° W		S/N 57° W		S/N 65° W		S/N 65° W		S/N 56° W
8										
12		N/S 56° E		N/S 52° E		N/S 48° E		N/S 42° E		N/S 40° E
16	3		9		15		21	N/S 22° W		N/S 25° W
20										
24		S/N 34° E		S/N 30° E		S/N 27° E		S/N 24° E		S/N 21° E
4		S/N 41° W		S/N 44° W		S/N 47° W		S/N 51° E		S/N 54° W
8										
12	4	N/S 84° E	10	N/S 67° E	16	N/S 67° E	22	N/S 62° E	28	N/S 54° E
16										
20										
24		S/N 45° E		S/N 41° E		S/N 36° E				
4		S/N 30° W		S/N 33° W		S/N 34° W		S/N 33° E		S/N 29° E
8	5		11		17		23	S/N 38° W	29	S/N 40° W
12		N/S 67° W		N/S 76° W		N/S 68° W		N/S 84° W		N/S 73° E
16										
20										
24										
4	6	S/N 59° E	12	S/N 55° E	18	S/N 49° E	24	S/N 45° E	30	S/N 40° E
8		S/N 22° W		S/N 23° W		S/N 25° E		S/N 27° W		S/N 29° W
12		N/S 47° W		N/S 55° W		N/S 61° W		N/S 63° W		N/S 76° W
16										
20										
24										
4		S/N 79° E		S/N 68° E		S/N 67° E		S/N 57° E		S/N 55° E
8										S/N 20° W
12		N/S 26° E		N/S 23° E		N/S 22° E				
16		N/S 35° W		N/S 37° W		N/S 44° W		N/S 48° W		N/S 54° W
20										
24										

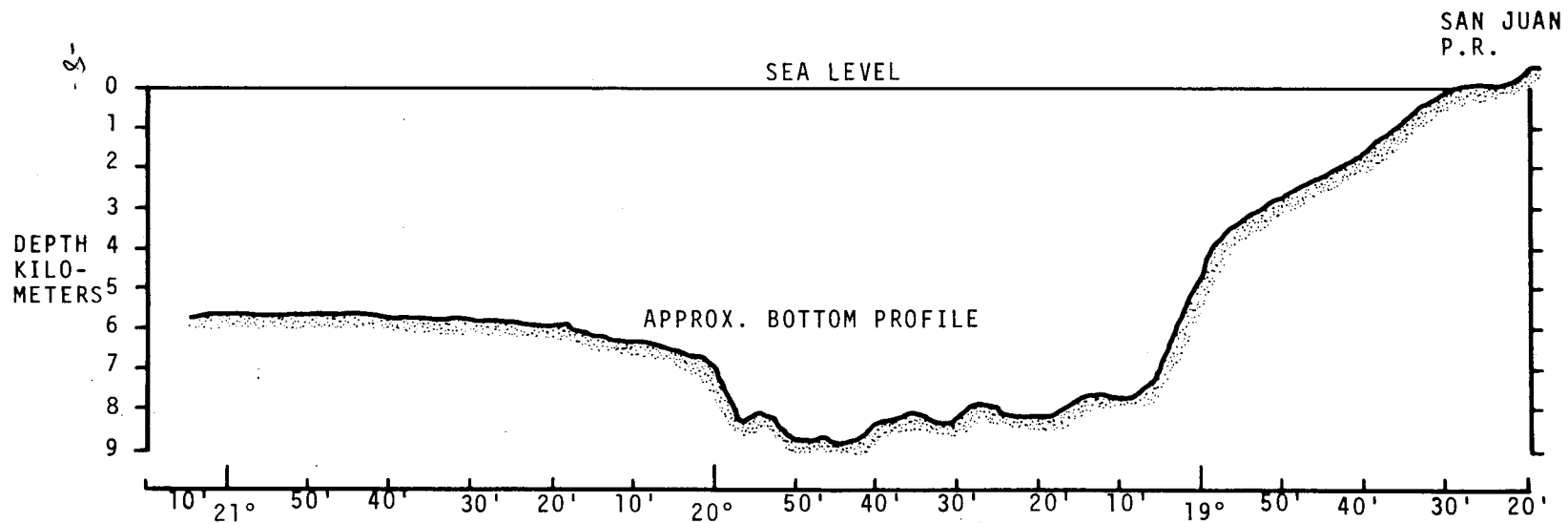
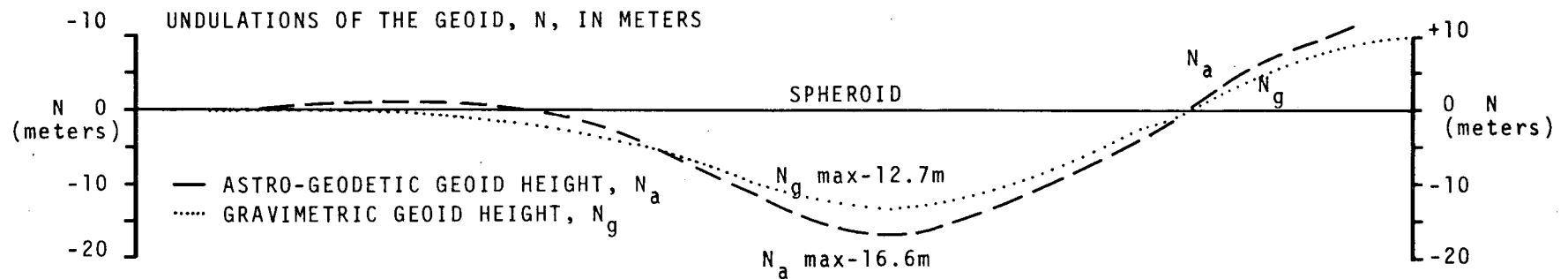
Table 3

Puerto Rico Trench Tracking Schedule For July 1970



PUERTO RICAN TRENCH PROFILE
ALONG MERIDIAN 66°3 WEST

Figure 2



(TOP) Curvature of geoid relative to spheroid.
(BOTTOM) Water depths of the Puerto Rico Trench.

SECTION 2.0

MEASUREMENT MODEL

For land based multi-station radar solutions, the orbit is determined almost completely by range data since the angle data provides substantially less accuracy at satellite ranges [2]. Thus the measurement model used in this analysis consists of slant range data only for all stations. The ship angle data will also be ignored for basically the same reasons. The epoch elements used in ORAN were simulated GEOS-II orbit elements and are representative of a typical GEOS-II orbit.

The measurement noise values used for all ORAN simulations are listed in Table 4 and represent conservative estimates of the capabilities of these instruments. The range sigmas for the land stations have been observed to exhibit typically a range noise of 0.5 meter to 1.0 meter at a sampling frequency of one point per second. The range sigmas for the ship are, of course, higher, and have been observed to be ~ 3 meters at a sampling rate of one point per second.

TABLE 4

RANGE MEASUREMENT STANDARD DEVIATIONS

<u>Station and Measurement Type</u>	<u>One-Sigma Uncertainty</u>
Land-Based Range	1.5 meters
Ship-Based Range	3.0 meters

Measurements were assumed to exist down to an elevation angle of 10 degrees. No measurements were allowed below this angle because of the difficulty in accurately correcting observed range data for tropospheric refraction in this region.

The a priori one sigma uncertainties on the ship position were set at 100 meters Y and X (latitude and longitude respectively) and 25 meters in Z (height) in the local coordinate system for all ORAN simulations.

SECTION 3.0

SOURCES OF UNMODELED ERRORS

The ORAN Program has the capability to investigate the effects of incomplete or non-existent corrections for certain systematic errors on the measurements used in an orbital data reduction. Biases are the most obvious - and in most cases, the most important - example of an error of this type. In addition to true measurement errors, actual orbit determinations are affected by two other major sources of error - station position errors and force field errors. Station position errors enter the calculated measurement directly and force field errors enter because the satellite position at any time past epoch depends upon the force field used in integrating the orbit. The effects on orbital data reductions of station position errors and certain force field errors can also be calculated by the ORAN program.

A reasonable best estimate or an upper limit for the expected error (or standard deviation) of the parameter is required to determine if it is a significant source of error in an orbital solution. Table 5 lists the uncertainty estimates for the set of parameters which have been considered for each tracking station used in the ORAN simulations discussed in this paper. In general, the values chosen are close to being upper limits for expected errors. An indication of the sources of these unmodeled errors and the procedures used for obtaining the magnitudes as listed in Table 5 are discussed in the following sections.

TABLE 5
SOURCES OF UNMODELED ERRORS AND THEIR MAGNITUDES

<u>Unmodeled Errors</u>	<u>Magnitude</u>
Range Bias (Ship and Land Based Multi-Stations)	5 meters
Refraction Error (Land Based Radars)	5%
Refraction Error (Ship)	10%
Center of Mass Error (X,Y,Z)	15 meters in each coordinate
Local Survey Error (X,Y,Z)	3 meters in each coordinate
Geopotential Coefficient Errors (SAO-APL Differences)*	100%
Gravitational Coefficient Error in $\mu = GM$	1 ppm
Gravitational Coefficient Error in Resonance Terms C(14,13) and S(14,13)	3.7236×10^{-20}

* Differences between the truncated (to 12th degree zonals and 8th order tesserals) Smithsonian Astrophysical Observatory (SAO) M1 model [3,4] and the truncated APL model.

3.1 INSTRUMENTATION ERRORS

3.1.1 Bias Errors

Errors of basically a bias type have been generally found to be the largest contributor to systematic error in range tracking systems. An estimate of instrumentation bias errors can be obtained from pre- and post-mission calibrations. The range bias can be found by comparing a series of measured ranges to a fixed ground target with the surveyed range value. Similarly, the biases in azimuth and elevation angles can be determined by comparing a series of angle measurements to a boresight tower in both normal and plunge operation. The mean difference between the measured range and the surveyed range is a measure of the bias in the range measurement and can be used accordingly as pre-processing correction for range. Unfortunately, the calibration measurements must normally be made at a very low elevation angle within the atmosphere and in the near field pattern of the antenna; since multipath and tropospheric refraction effects on the calibration process are both somewhat uncertain and variable, the residual biases after calibration may be considerable.

3.1.2 Refraction Errors

The tropospheric refraction correction based upon the ray path integration using a measured vertical refractive index profile has an expected error on the order of 2 - 4% [5]. The use of a correction procedure based upon a surface index only should introduce a few percent additional errors. Taking into account the fact that the location of the land-based radars near the

land-sea boundary where atmospheric conditions are quite difficult to predict, a residual refraction error of 5% of the correction is an approximate upper limit to the error which could be expected; the ship refraction correction is more difficult and its error is taken as twice the land value or 10%.

3.2 STATION POSITION ERRORS

The orbit prediction process predicts the position of a satellite in an inertial coordinate system by relating the satellite to known points in a local (radar-centered) coordinate system. Consequently any error in the position of the reference points will degrade the prediction accuracy. The best determinations from satellite motion have recovered station positions to 15 meters with respect to the geocenter [3]. In fact references [6] and [7] state that the center of mass coordinates of the Smithsonian Astrophysical Observatory (SAO) C-5 Baker-Nunn stations have been assessed to have approximately 15 - 20 meter accuracy.

For the purposes of this study, the uncertainties in the X, Y, Z position of center of mass coordinates have been assumed at 15 meters each, while 3 meters has been assumed for the position uncertainty for each local X, Y, Z position with respect to Antigua.

Since all stations utilized in this study are positioned on the SAO C-5 Datum (See Table 1), they all have a comparable positional uncertainty with respect to the geocenter. For simplicity, all stations were also assumed to have the same topocentric position uncertainty.

3.3 FORCE FIELD ERRORS

At the altitude of the GEOS-II satellite, atmospheric drag forces are negligible. Since the perturbations due to the sun and moon can be quite accurately modeled in the orbit generation process, the only significant force field errors are those in the earth's gravitational coefficient and its harmonic coefficients.

3.3.1 Gravitational Coefficient

The best estimate of the gravitational constant, GM_o , has been obtained from the reduction and analysis of Ranger lunar radio tracking data by the Jet Propulsion Laboratory. The uncertainty in their determination is $\pm 1 \times 10^{-6}$ [8]. This value was carried as an unmodeled error in all simulations.

3.3.2 Geopotential Coefficients

Because significant variations exist for the geopotential coefficients recovered from terrestrial and/or satellite tracking data - differences which are generally much greater than the quoted standard deviations - a scheme has been adopted which utilizes any chosen percentage of the difference between any two of the best determined gravity models as the effective uncertainty in the total set of gravitational harmonics. In this manner, any fractions of this difference can be represented in the solution as an unmodeled error.

Assuming the existence of more than one gravity model of comparable accuracy and that these models do not have too much common ancestry, this appears to be

the most valid representation of the total geopotential coefficient errors. Among the most accurate unclassified geopotential models are the Smithsonian Astrophysical Observatory (SAO) M1, the Applied Physics Laboratory (APL) 3.5, and the Naval Weapons Laboratory (NWL) 5E-6. Differences between any two of these and several other models are available in the ORAN program as a representation of the geopotential coefficient error. As listed in Table 5 the ORAN runs considered in this paper used 100% of the differences between the truncated SAO M1 and APL 3.5 models as the geopotential coefficient error.

SECTION 4.0

CHOICE OF TRACKING STATIONS

There are a total of 8 tracking stations that were considered in this study including possibility of the lasers at Curacao and Puerto Rico and the ship at various locations across the Trench. Of these 8 stations, it was found that the lasers at these sites were unnecessary because they contributed little to the orbit determination and height error recovery solutions. To investigate the effects of various combinations of the remaining tracking stations on the orbit estimation and height recovery capability, computations were made using a network of 6 stations, 4 stations, and 3 station tracking complements. The results are presented in Tables 6 and 7. Table 6 shows a comparison of the average orbit uncertainties as a function of the tracking complement. Table 7 shows a comparison of the uncertainties in the recovered ship Z coordinate (altitude) due to noise, noise plus all unmodeled error effects and a 5 meter ship range bias on the 6 station and 4 station tracking complement studies.

From an orbit recovery viewpoint and using a single pass solution, the 6 station solution is superior to both the 4 station and 3 station solutions by a factor of 3:1; however, the differences between the solutions are not significant enough to require a tracking capability of 6 stations. In addition, it should be noted that there is little significant difference in the estimate of ship height error, σ_z , between the 6 station tracking solution and the 4 station tracking solution. Thus, σ_z within this study range is relatively independent of the number of tracking stations. Furthermore, it was found that the error in ship height (σ_z) is also independent of the ship's location along the Puerto Rico Trench.

TOTAL (AVERAGE) UNCERTAINTY	SIX STATION COMPLEMENT (ANT/GRT/WAL/MRT/BER/SHIP)		FOUR STATION COMPLEMENT (ANT/GRT/BER/SHIP)		THREE STATION COMPLEMENT (ANT/GRT/SHIP)	
	HIGH ELEVATION	LOW ELEVATION	HIGH ELEVATION	LOW ELEVATION	HIGH ELEVATION	LOW ELEVATION
σ_R (m)	0.39	0.50	1.33	1.87	1.34	1.85
σ_V (cm/s)	0.05	0.19	0.21	0.30	0.21	0.30

TABLE 6
AVERAGE ORBIT UNCERTAINTIES (SINGLE PASS SOLUTIONS)

Arc. No.	1		2		3		4		5		6		7		8		9	
Maximum Elevation (deg)	88.8		87.7		89.2		86.4		88.6		85.3		87.5		47.4		50.0	
Pass Relation to Ship	West		West		East		West		East		West		East		West		East	
No. of Stations Used in Tracking Complement	6	4	6	4	6	4	6	4	6	4	6	4	6	4	6	4	6	4
σ_z of Ship (meters) Noise Only	0.53	0.54	1.99	2.03	0.22	0.23	4.30	4.35	0.48	0.49	3.66	3.74	0.67	0.68	1.87	1.92	1.57	1.61
σ_z of Ship (meters) Noise & Unmodeled Errors	16.65	18.75	34.69	38.88	6.64	6.57	18.95	14.85	5.41	2.34	54.93	53.91	7.88	5.53	24.72	22.92	19.41	18.94
σ_z of Ship (meters) 5m Range Bias	14.91	14.86	30.98	30.54	5.01	5.02	-4.9	-4.60	0.5	-0.05	-43.50	-42.81	-3.94	-3.93	-17.50	-17.54	-14.40	-14.46

TABLE 7
VARIATION IN σ_z (SHIP) WITH TRACKING COMPLEMENT

Utilizing the results and conclusions of the above investigations, a detailed analysis of the three station solution was initiated. In addition, the somewhat pessimistic results indicated to this point concerning the estimation of ship height using single satellite passes indicated that other approaches such as multiple satellite pass solutions should also be explored. The results of these further investigations are presented in the following sections.

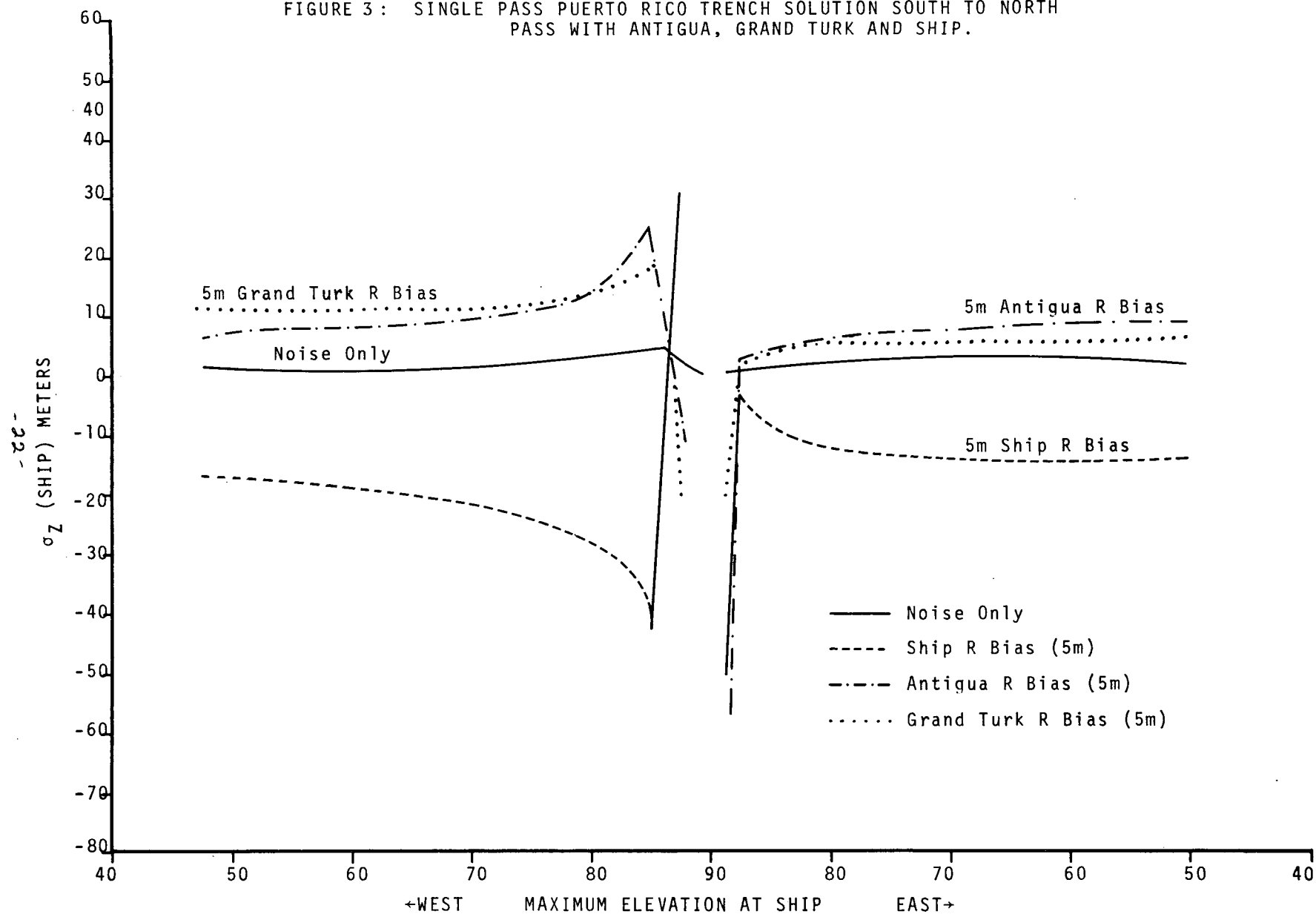
SECTION 5.0

SHIP HEIGHT ESTIMATION USING ONE SATELLITE PASS

Since we have found that neither the 6 station nor the 4 station tracking complement is necessary we will now investigate more fully the effects of single pass range-only solutions on the errors in ship height, σ_z , due to various unmodeled parameters (see Section 3) for the three station tracking complement of Antigua, Grand Turk and the Ship. Figure 3 shows the variation in σ_z with maximum elevation angle on both sides of the ship for a south to north track of the satellite. The region between $85^\circ < E < 90^\circ$ (East) and $85^\circ < E < 90^\circ$ (West) contains an inherent singularity in the covariance matrix due to simultaneously solving for all three components (latitude, longitude and height) of the ship's position. Computations have been made to show that this ill-conditioning problem disappears when only ship height (σ_z) is solved for.

It is apparent from the results of the single pass solutions that rather large amplitudes exist in the σ_z of the ship. For example, the noise only solution (ignoring unmodeled error effects) gives a bias in σ_z between 1 and 2 meters on either side of the ship. This error is very large for a measurement noise contribution. In addition, the effects of a 5 meter range bias in each of the tracking stations results in a bias in σ_z between 6 meters and 20 meters. Large differences exist even for passes on different sides of the ship. Furthermore, from the previous study results, it does not appear that these errors can be reduced appreciably by including additional tracking stations.

FIGURE 3: SINGLE PASS PUERTO RICO TRENCH SOLUTION SOUTH TO NORTH
PASS WITH ANTIGUA, GRAND TURK AND SHIP.



It is clear that maximum elevation angles between 50° and 70° present the most optimum relative profile recovery condition utilizing single pass solutions. However, this geometry is available only once during any 6 to 10 day period for the GEOS-II satellite. Therefore, in an effort to eliminate this constraint on elevation angle and experiment duration the capabilities of the two pass solution were investigated. The results are presented in the following section.

SECTION 6.0

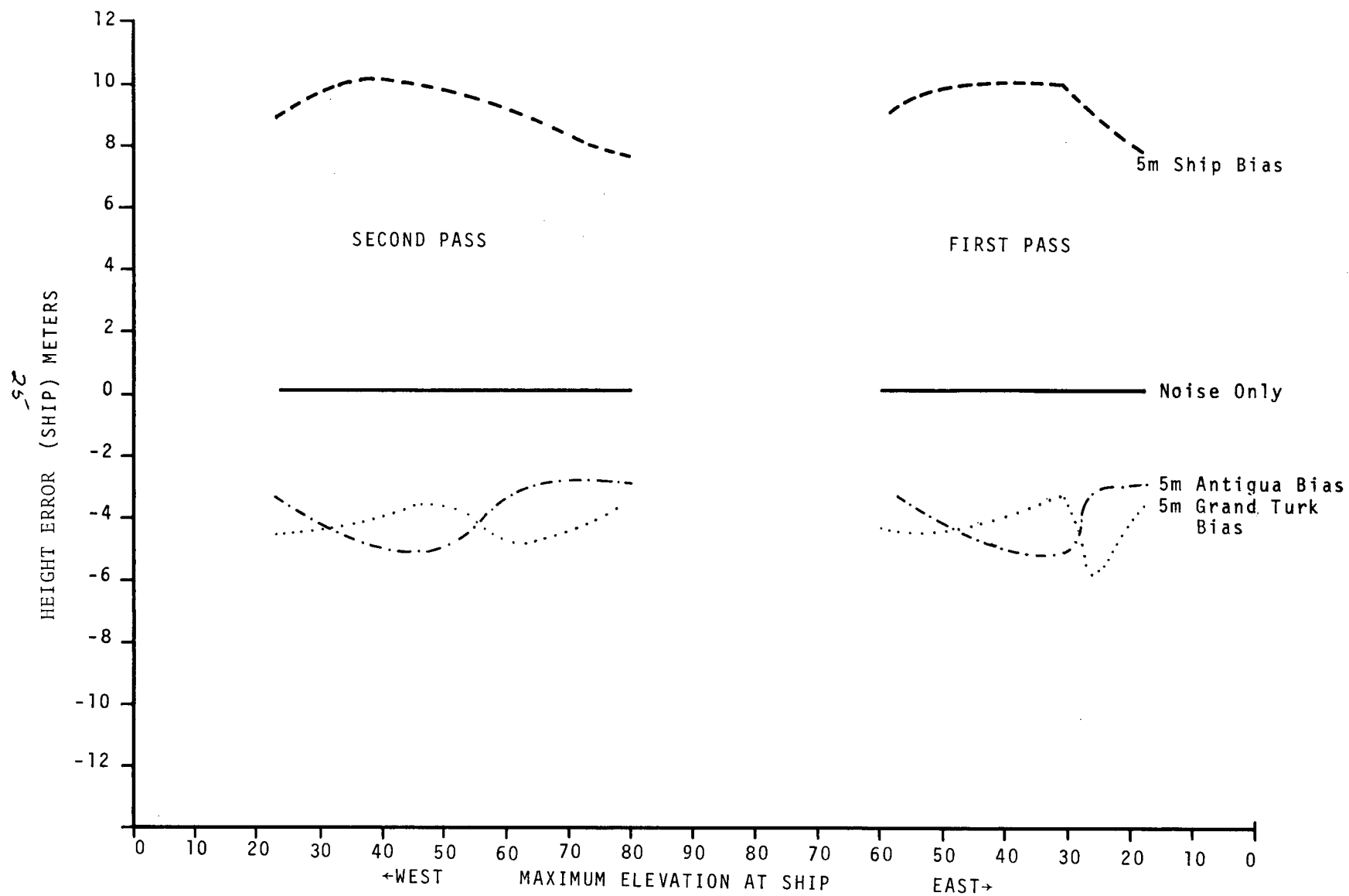
SHIP HEIGHT ESTIMATION USING TWO SATELLITE PASSES

As has been shown in the previous section, single pass solutions for the ship height are quite sensitive to systematic measurement errors and to pass geometry. To evaluate means of reducing this sensitivity, a series of two pass ORAN simulations were made assuming range measurements from Antigua, Grand Turk and the Ship.

The results of these simulations are summarized in Figure 4, in which the effects of the most significant measurement errors on recovered ship height are shown as a function of maximum ship elevation angle. Since there are two passes for the ship, the effects on ship height are presented twice, once as a function of maximum elevation angle for the satellite pass east of the ship and once for passes west of the ship. Five meter radar biases are seen to have an effect which has a variation of only two meters as the satellite pass geometry is completely exercised over a full 6 day cycle. Thus, determination of the relative geoid profile appears feasible with two meters resolution utilizing this approach.

Although the 5 meter radar bias levels are pessimistic values compared to the calibrations expected of both the ground based and ship-based radars, it is of interest to note that biases of the same value at the two ground stations tend to produce a total effect which is more nearly constant than is the effect of either alone. That is, the addition of the bottom curves in Figure 4 produces curves that are nearly flat, except for sets of passes which contain a low elevation pass ($\sim 25^\circ$) east of the ship. This suggests that if low elevation passes must be used, they should be west of the ship.

FIGURE 4 . COMBINED TWO PASS SOLUTION SOUTH TO NORTH
PASS FOR ANTIGUA, GRAND TURK AND SHIP



One other consideration, which is inherent in the two-pass-solution approach (as well as the single pass solution) is the requirement that the ship either remain stationary during the entire two pass time period or adequate knowledge of the ship position changes be available for later data reductions. The study of methods of meeting this requirement and any inherent problems is considered outside the scope of this study and are to be considered in a follow on analysis.

SECTION 7.0

CONCLUSIONS

In this report, we have attempted to show the effects of several unmodeled parameters on various multi-station tracking situations. From the results of the investigation we have arrived at the following general conclusions:

- 1) Antigua and Grand Turk provide satisfactory orbits for South-to-North type passes. Merritt Island, Bermuda and possibly Wallops provide secondary (back-up) orbits on North-to-South type passes.
- 2) High elevation single pass solutions are necessary to give moderate sensitivity to systematic error effects. These solutions have disadvantages of:
 - a) Moderate sensitivity to elevation differences on various passes.
 - b) Only one usable pass available during any 6 - 10 day period.
 - c) Large differences for passes on different sides of the ship.
- 3) Two pass solutions provide reduced sensitivity to exact pass geometries. The most critical factors are that:
 - a) Ground station and ship range biases must be constant.

- b) Ship motion between passes must be accounted for much better than the ships inertial navigation system (SINS) normally does.
- 4) Two relative profiles can be obtained - one from South-North type satellite passes and one from North-South type satellite passes. There is a possibility that these profiles might be tied together with two satellite passes while the ship is docked in San Juan, Puerto Rico.
- 5) The error in ship height, σ_z , is relatively independent of the location of the ship along the Trench.

The optimum tracking schedule for the time period of 1 June through 30 July 1970 is listed in Table 8 and only lists the usable South-to-North two pass combinations.

OPTIMUM TRACKING SCHEDULE

<u>Day</u>	<u>Hour</u>	<u>Max El</u>	<u>Day</u>	<u>Hour</u>	<u>Max El</u>
5/1	2400	40°	5/28	2400	66°
5/2	0200	38	5/29	0200	20
5/2	2400	53	6/1	2400	25
5/3	0200	28	6/2	0200	55
5/3	2400	69	6/2	2400	34
5/4	0200	20	6/3	0200	41
5/6	2300	28	6/3	2400	45
5/7	0100	54	6/4	0200	30
5/7	2400	38	6/5	0000	59
5/8	0200	41	6/5	0200	22
5/8	2400	49	6/8	2400	30
5/9	0200	30	6/9	0200	44
5/9	2400	61	6/9	2400	41
5/10	0200	22	6/10	0200	33
5/12	2400	26	6/11	0000	55
5/13	0200	57	6/11	0200	23
5/13	2400	34	6/14	2400	27
5/14	0200	44	6/15	0200	47
5/14	2400	45	6/15	2400	36
5/15	0200	33	6/16	0200	34
5/15	2400	59	6/17	0000	49
5/16	0200	24	6/17	0200	25
5/19	2400	31	6/22	0000	33
5/20	0200	48	6/22	0200	38
5/20	2400	42	6/23	0000	45
5/21	0200	35	6/23	0200	27
5/21	2400	53	6/28	0000	29
5/22	0200	26	6/28	0200	40
5/25	2400	28	2/29	0000	40
5/26	0200	50	6/29	0200	29
5/26	2400	37	6/30	0000	55
5/27	0200	38	6/30	0200	20
5/27	2400	50			
5/28	0200	28			

TABLE 8

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